A Hoare Logic for Domain Specification

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- Abstraction of computational details from other modules
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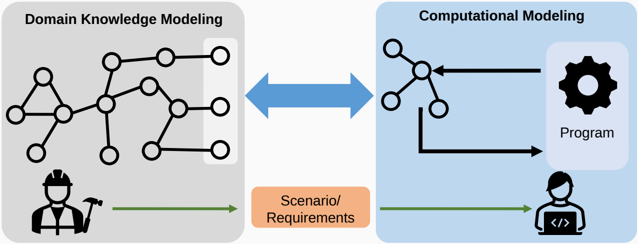

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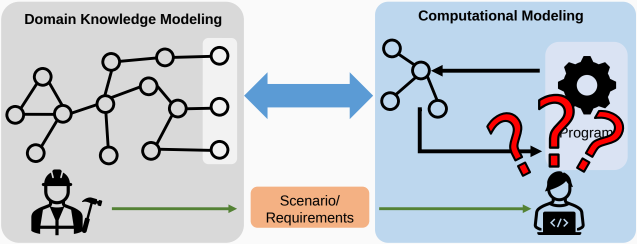

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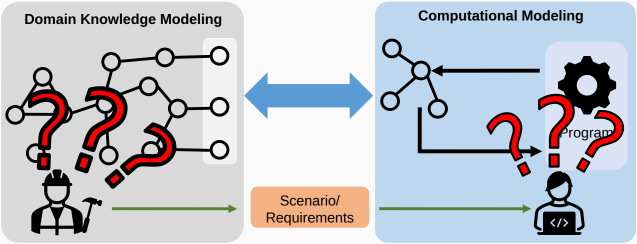

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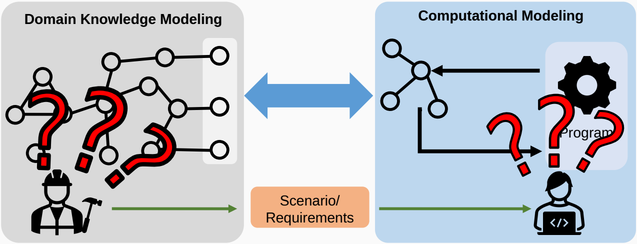

- Domain bugs are hard to find *and express*
- How can we use pragmatics of domain modeling tools inside a proof?
- How can we manage domain and computational specification during proofs?

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Suppose you model the assembly process of a car

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This procedure sets the number of wheels in a car to the value of p.

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Subject Matter Expert

I want that in the end of this step, the car classifies as a small car.

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How to enable both of them to specify their respective intent?

- SME does not know about how the car c is encoded
- Programmer does not know what it means for a car to be small.

Lifted Specification

Ontologies and Description Logics

For domain modeling and specification a rich body of methodologies and tools exist.

$$\texttt{HasFourWheels} \sqsubseteq \texttt{Small} \qquad \exists\texttt{wheels}.\exists\texttt{hasValue}.4 \equiv \texttt{HasFourWheels}$$

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$$\begin{Bmatrix} - \\ \texttt{p} \doteq 4 \end{Bmatrix} \texttt{addWheels}(\texttt{p}) \begin{Bmatrix} \texttt{Small}(c) \\ - \end{Bmatrix}$$

- Upper component specifies the state as interpreted in the domain
- Lower component specifies non-lifted state

Keeping State and Lifted State Connected

Idea: define a compatible lifting of the specification as well.

$$\left\{ \qquad\qquad\qquad \right\}$$

$$\texttt{nrWheels} := \texttt{p}$$

$$\left\{ \texttt{Small}(c) \qquad\qquad\qquad\qquad\qquad\qquad \right\}$$

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Perform following steps for wp reasoning:

1. Infer (abduct/deduct) lifted post-condition
2. Recover state post-condition, substitution
3. Lift pre-condition, deduce domain pre-conditions

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$$\texttt{nrWheels} := \texttt{p}$$

$$\left\{ \texttt{Small}(c), \texttt{HasFourWheels}(c), \texttt{hasValue}(\texttt{wheelsVar}, 4) \right\}$$

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Keeping State and Lifted State Connected

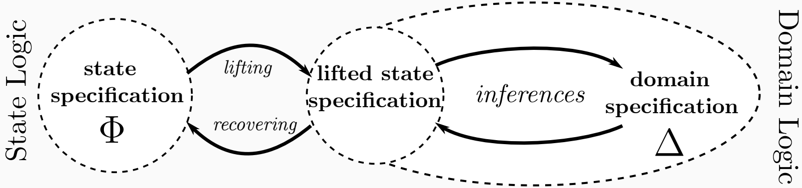
State Lifting

Function μ from runtime states to knowledge graphs.

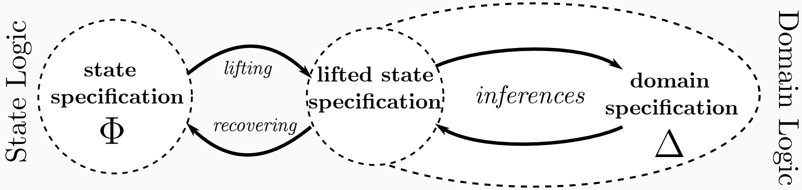
Specification Lifting

Function $\widehat{\mu}$ from program assertions to axioms. Must be compatible to state lifting:

$$\sigma \models \phi \rightarrow \mu(\sigma) \models \widehat{\mu}(\phi)$$



A Signature Perspective



Kernel and Generator

Let Σ be the signature of the domain specification.

- The kernel of $\widehat{\mu}$ is a signature **ker** $\widehat{\mu} \subseteq \Sigma$.
- A core generator α maps axioms Δ to axioms $\alpha(\Delta)$ with $\alpha(\Delta) \models \Delta$

- Kernel generator can either implement deduction, or abduction
- In case of abduction: ABox abduction with signature abducibles

Some Rules

- First you generate the kernel
- Additional premise trivial if α is deductive

$$\text{pre-core}\ \frac{\Delta_2 \models^{\mathbf{K}} \alpha(\Delta_2) \qquad \mathbf{K} \vdash \left\{{\Delta_1 \atop \Phi_1}\right\} s \left\{{\Delta_2, \alpha(\Delta_2) \atop \Phi_2}\right\}}{\mathbf{K} \vdash \left\{{\Delta_1 \atop \Phi_1}\right\} s \left\{{\Delta_2 \atop \Phi_2}\right\}}$$

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- Same for precondition
- On state assertions, we can now use standard Hoare rules

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A Car is a `Car`

- Standard Hoare calculus rules must check that specifications are consistent, and
- remove all domain knowledge, as it may have changed

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But now, we can prove that our program does the right thing:

$$\cfrac{\cfrac{\cfrac{\cfrac{}{\mathtt{hasValue}(\mathtt{wheelsVar}, 4) \models^{\mathbf{K}} \mathtt{HasFourWheels}(c), \mathtt{hasValue}(\mathtt{wheelsVar}, 4)}}{\mathbf{K} \vdash \left\{{- \atop \mathtt{p} \doteq 4}\right\}\mathtt{nrWheels} := \mathtt{p}\left\{{\mathtt{HasFourWheels}(c), \mathtt{hasValue}(\mathtt{wheelsVar}, 4) \atop \mathtt{nrWheels} \doteq 4}\right\}}}{\mathbf{K} \vdash \left\{{- \atop \mathtt{p} \doteq 4}\right\}\mathtt{nrWheels} := \mathtt{p}\left\{{\mathtt{HasFourWheels}(c), \mathtt{hasValue}(\mathtt{wheelsVar}, 4) \atop -}\right\}}}{\mathbf{K} \vdash \left\{{- \atop \mathtt{p} \doteq 4}\right\}\mathtt{nrWheels} := \mathtt{p}\left\{{\mathtt{HasFourWheels}(c) \atop -}\right\}}$$

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Thank you for your attention