

Type-Based Verification of Delegated Control in Hybrid Systems

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Modern Cyber-Physical Systems require Distributed Control and Cloud Systems

- Edge devices in IoT
- Digital Twins and Industry 4.0
- Networked devices, e.g., autonomous trains



Engineers can build these devices – but how do we verify them?
CPS verification, program verification and cloud modeling barely intersect.

Modeling - Hybrid Active Objects



Abstract *Behavioral* Specification

(1) Modeling, (2) Specification and Verification, and (3) Simulation of Modular Systems with Active Objects.

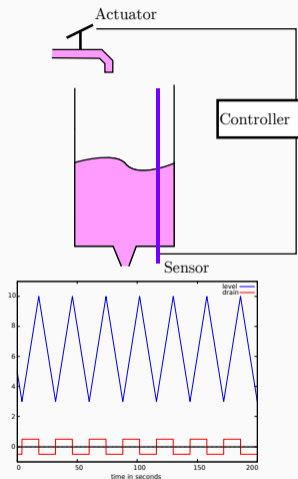
Active Objects = objects
 + actor concurrency model
 + condition synchronization
 + explicit time

Abstract *Behavioral Specification*

(1) Modeling, (2) Specification and Verification, and (3) Simulation of Modular Hybrid Systems with Active Objects.

Hybrid Active Objects = objects
 + actor concurrency model
 + condition synchronization
 + explicit time
 + continuous behavior

Example: Water Tank



```
class CSingleTank(Real inVal){  
  physical{  
    Real lvl = inVal : lvl' = flow;  
    Real flow = -0.5 : flow' = 0;  
  }  
  { this!low(); }  
  Unit low(){  
    await diff lvl <= 3 & flow <= 0;  
    flow = 0.5; this!up();  
  }  
  Unit up(){  
    await diff lvl >= 10 & flow >= 0;  
    flow = -0.5; this!low();  
  }  
}
```

Is $3 \leq lvl \leq 10$ an invariant (if $3 \leq inVal \leq 10$)?

Verification - Post-Regions

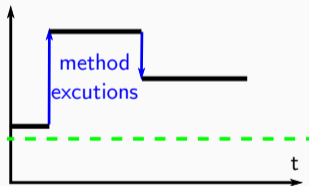


Proof Obligations with Dynamic Logic

In discrete systems, an object invariant I can be checked *modularly* with dynamic logic by showing that every method preserves I .

$$I \rightarrow [s]I$$

Proof Obligation for Discrete Systems



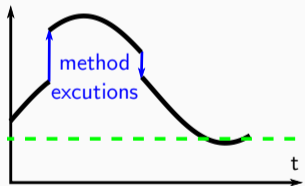
First, we need a logic for hybrid systems.

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Proof Obligation for Discrete Systems



First, we need a logic for hybrid systems.

Differential Dynamic Logic

A logic for (algebraic) hybrid programs:

$$\phi ::= \forall x. \phi \mid \phi \vee \phi \mid \neg \phi \mid \dots \mid [\alpha] \phi$$

$$\alpha ::= ?\phi \mid \mathbf{v} := t \mid \mathbf{v} := * \mid \{\mathbf{v}' = f(\mathbf{v}) \& \phi\} \mid \dots$$

Differential Dynamic Logic

A logic for (algebraic) hybrid programs:

$$\begin{aligned}\phi &::= \forall x. \phi \mid \phi \vee \phi \mid \neg \phi \mid \dots \mid [\alpha] \phi \\ \alpha &::= ?\phi \mid v := t \mid v := * \mid \{v' = f(v) \& \phi\} \mid \dots\end{aligned}$$

Example

Set a variable to 0, let it raise with slope 1 while it is below 5 and discard all runs where it is above 5.

$$[x := 0; \{x' = 1 \& x \leq 5\}; ?x \geq 5] x \dot{=} 5$$

This formula is valid.

Preliminaries

- We assume that every method starts with an **await diff** statement.
If it does not, add **await diff true**.
- The leading guard of a method m is denoted $trig_m$.
- Only **Real** variables are manipulated.
- Weak negation is denoted $\neg e_1 \geq e_2 \iff e_1 \leq e_2$

Safety

An object is safe w.r.t. some formula ϕ , if its state is a model for ϕ
(a) whenever a method starts and (b) whenever time advances.

For the beginning, we assume that all **await** are leading and no **get** or **duration** occur.

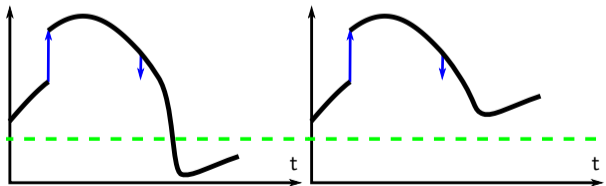
Theorem

Let C be a class with dynamics ode . Each object of C is safe w.r.t. inv and precondition pre if for every method the following holds:

$$inv \rightarrow [?trig_m; trans(s_m)] (inv \wedge [ode \& true] inv)$$

And additionally for the constructor:

$$pre \rightarrow [trans(s_{init})] (inv \wedge [ode \& true] inv)$$



Lemma

Let C be safe w.r.t. inv . Let C^+ be C with an added method and C^- be C with a method removed.

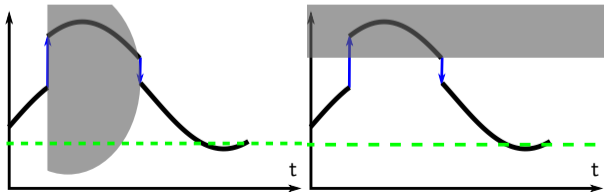
- C^- is safe
 - To show safety of C^+ , only the new method must be verified
-
- Very modular
 - Imprecise: do not use additional information provided the structure
 - Cannot verify our water tank
 - Can verify self-stabilizing systems without control cycle

Theorem

Let \mathcal{C} be a class with dynamics ode . For each method m let CM_m be the set of methods which are guaranteed to be called in every execution. Each object of \mathcal{C} is safe w.r.t. inv if for every method m the following holds:

$$\text{inv} \rightarrow [\text{?trig}_m; \text{trans}(s_m)] \left(\text{inv} \wedge \left[\text{ode} \& \bigwedge_{m' \in \text{CM}_m} \text{?trig}_{m'} \right] \text{inv} \right)$$

And analogously for the constructor.



```
class LocalTank(){  
  physical{Real lvl = 5 : lvl' = flow; Real flow = -0.5 : ...}  
  { this!low(); }  
  Unit low(){await diff lvl <= 3; flow = 0.5; this!up();}  
  Unit up(){await diff lvl >= 10; flow = -0.5; this!low();}  
}
```

$$\text{inv} \rightarrow [?lvl \leq 3; \text{flow} := 0.5](\text{inv} \wedge [lvl' = \text{flow} \& lvl \leq 10]\text{inv})$$

```
class LocalTank(){  
  physical{Real lvl = 5 : lvl' = flow; Real flow = -0.5 : ...}  
  { this!timed(); }  
  Unit timed(){  
    await duration(1,1);  
    if(lvl >= 9.5) -flow = 0.5;  
    if(lvl <= 3.5) flow = 0.5;  
    this!timed();  
  }  
}
```

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class LocalTank(){  
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    this!timed();  
  }  
}
```

What about systems that decouple control and dynamics?

```
class Tank(Real inVal){                               /*@ requires 3.5 <=inVal<= 9.5 @*/  
  physical{ Real lvl' = flow; ...} /*@ invariant 3 <=lvl<= 10 && -0.5 <=flow<= 0.5 @*/  
  Unit localCtrl(){  
    if(lvl <= 3.5) flow = 0.5;  
    if(lvl >= 9.5) flow = -0.5;}}
```

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```

Need to consider other objects to compute post-regions – is the tank controlled?

```
class Controller(){  
  Unit timer(Tank t, Int time){  
    await duration(1);  
    if(time != 0) {  
      t!localCtrl();  
      this.timer(t, time - 1);}}}
```

```
class Tank(Real inVal){                               /*@ requires 3.5 <=inVal<= 9.5 @*/  
  physical{ Real lvl' = flow; ...} /*@ invariant 3 <=lvl<= 10 && -0.5 <=flow<= 0.5 @*/  
  Unit localCtrl(){  
    if(lvl <= 3.5) flow = 0.5;  
    if(lvl >= 9.5) flow = -0.5;}}
```

Need to consider other objects to compute post-regions – is there always one controller?

```
class Mobile {  
  Unit run() {  
    Tank t = new Tank(4);  
    Controller c = new localCtrl(); Fut<Unit> f = c.timer(t, 40);  
    await duration(40) & f;  
    c = new Controller(); f = c.timer(t, -1); }}
```

Subtle timing issues can violate specification

```
class Controller(){
    Unit timer(Tank t, Int time){
        await duration(1);
        if(time != 0) {
            t!localCtrl();
            this.timer(t, time - 1);}}}

class Mobile {
    Unit run() {
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        await duration(40) & f;
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```

External Control



Challenge

- Post-region cannot be computed locally
- External control must be globally ensured
- Obligation for external control can be delegated

Overview Solution

- Controllee gets specification: Temporal, externally controlled post-region (ECP)
- Controller gets specification: controlled objects
- Type system checks
 - For every object with an ECP there is always a controller
 - Each controller respects the ECP specification of its controllee
- From the controllee-view, post-region-based verification is unchanged

```
class Tank(Real inVal){                               /*@ requires 3.5 <=inVal<= 9.5 @*/  
  physical{ Real lvl' = flow; ...} /*@ invariant 3 <=lvl<= 10 && -0.5 <=flow<= 0.5 @*/  
  /*@ timed_requires 1 @*/  
  Unit localCtrl(){  
    if(lvl <= 3.5) flow = 0.5;  
    if(lvl >= 9.5) flow = -0.5;}}  
  
class Controller(){  
  /*@ time_control: t.localCtrl = [1, 1] @*/  
  Unit timer(Tank t, Int time){
```

- `timed_requires` specifies the period of repeated calls to this method
- `timed_control p.m= [a,b]` specifies what the method periodically calls
 - Periodic call to `p.m` with some ECP, where `p` is a parameter and thus invariant
 - The first time after `a` time units
 - After the last call, `b` time units remain until it must be called again

ECP Analysis

Three step analysis

- Run a global time analysis, derive for each statement how much time it may require to execute it (non-modular, lightweight)
- Run type system, to make sure ECP are called correctly (non-modular, lightweight)
- Generate and verify all proof obligations with ddL/KeYmaera X (modular, heavyweight)

Type system operates on the level of locations.

- A ceid is a pair of location and method (e.g. p, m). Idea:
- Keep track of all ceid's and when it must be called again during type checking
- Update maximal time left for each ceid's to be called
- Check that this time is always positive
- Delegation only through method calls, i.e., tree like structure

$$\Gamma_l, \Gamma_d \vdash s : \Gamma'_l, \Gamma'_d$$

- Γ_l registers the ceid's that the method under analysis must control, maps to a number
- Γ_d registers the ceid's that we delegated control to and maps them to $(fid, t_{min}, t_{max}, t)$: Future fid , how long they control t_{min}, t_{max} and when control must be called afterwards t

Rule for time advance without calls (simplified)

$$\Gamma_I, \Gamma_d \vdash s : \Gamma'_I, \Gamma'_d$$

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$$TA(s) = [t^-, t^+] \quad C = \{i \mid i \in I \wedge t_{min}^i - t^+ < 0\}$$

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$$\Gamma'_I = \Gamma_I^1 \cup \Gamma_I^2 \quad \forall \text{ceid} \in \Gamma'_I. \Gamma'_I(\text{ceid}) \geq 0$$

$$\Gamma_I, [\text{ceid}_i \mapsto (fid^i, t_{min}^i, t_{max}^i, t^i)]_{i \in I} \vdash s : \Gamma'_I, \Gamma'_d$$

- Rule for method calls is responsible for two things:
 - If one delegates control, move $ceid$ from Γ_l to Γ_d
 - If the method has a ECP, update Γ_l
- When object is created, all its methods with ECP are added to Γ_l
- Full system needs context-awareness and some other rules (see paper)

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Rule for calls (simplified)

$$(e_1, m) \in \mathbf{dom} \Gamma_l$$

$$\Gamma'_l = \Gamma_l[(e_1, m) \mapsto \mathit{treq}(T_1.m)]$$

$$\Gamma_l, \Gamma_d \vdash e_1 !m(e_2, \dots, e_n)$$

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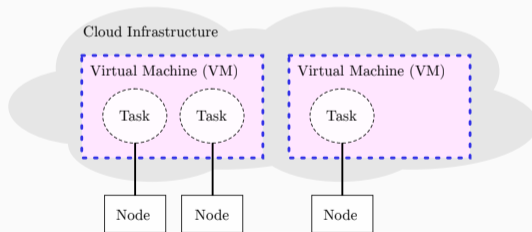
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 \end{array}$$

- Type system works for Timed ABS
- If model can be separated into timed control structure and hybrid, HABS can be used
- Cyber-physical systems are only at the edge!



Conclusion



Summary

- Post-regions for external control
- Type system ensures that control structures respect timing constraints
- Modular in time-analysis
- Modularity of deductive verification preserved

Future Work

- Implementation
- Beyond tree structured delegation
- Further post-region patterns

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Thank you for your attention